Advanced Algorithm

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Lecture 3: Balls and Bins (continue)

Balls and Bins

1. $m = o(\sqrt{n})$: $k = 1$ w.h.p;
2. $m = \Theta(\sqrt{n})$(Birthday Paradox): $k = 1$ or 2 w.h.p;
3. $m = n$: $k = \Theta(\frac{\ln n}{\ln \ln n})$ w.h.p;
4. $m \geq n \ln n$: ?

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1. $m = o(\sqrt{n})$: $k = 1$ w.h.p;
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4. $m \geq n \ln n$: $k = \Theta(\frac{m}{n})$ w.h.p.

Coupon Collector's Problem

- Randomized Algorithm - Chapter 3.6 (P57)
- Find m, such that $Pr(\min(X_1, \cdots, X_n) \geq 1) = 1 - o(1)$.

Key Points

- Markov Inequality
- Chebyshev's Inequality
- Chernoff's Bound
- $E(X_1 + \cdots + X_n) = E(X_1) + \cdots + E(X_n)$, no condition
- Union Bound: $Pr(A_1 \cup A_2 \cup \cdots \cup A_n) \leq Pr(A_1) + Pr(A_2) + \cdots + Pr(A_n)$, no condition

Lecture 4: Principle of Deferred Decisions

Principle of Deferred Decisions

- Ref: Randomized Algorithm - Chapter 3.5
- Poke game: Clock Solitaire
- Seating problem in the airplane
- Stable Marriage Problem

- "Men Propose" Algorithm
- "Deferred Acceptance with Compensation Chains" by Piotr Dworczak, EC 2016

Homework

1. In the coupon collector's problem, let T_i be the first step that there are exactly i non-empty boxes and let $T_0 = 0$. Let $Z_i = T_i - T_{i-1}$. We compute the expectation of Z_i in the class. In the homework, please compute the variance of Z_i.
2. Consider the case with n balls and n bins, let X be the random variable of the number of empty bins. Compute $E(X)$.
3. Estimate the deviation between X and $E(X)$ in the previous question. Your result should be in the form $Pr(|X - E(X)| > a) < b$.
4. Reference: Randomized Algorithm, chapter 4.4